Numerical Investigation of the Free Oscillations of an Inhomogeneous Axisymmetric Plasma Column

Dennis W. Hewett, May 1973
Lawrence Livermore National Laboratory, Livermore, CA

(Summary section from dissertation)

In the preceding chapters, techniques have been developed that allow one to numerically simulate an axisymmetric, inhomogeneous plasma column. The simulation is capable of carrying forward self–consistently all six components of the electric and magnetic field—neglecting only those purely electromagnetic effects. The simulation is based upon the electron Vlasov equation with coordinates $\rho, z, v_\rho, v_\phi, v_z$, and time; and, thus, the simulation retains the capability to deal with phenomena whose origin lies in the detailed shape of the velocity profile.

We believe that the simulation method presented here is presently the most economical method of realistic ρ, z simulation. The small size of the representation of physical quantities in this method, as compared to discrete particle multidimensional simulations, limits primarily only the length of time that meaningful solutions can be maintained. A typical run on a Honeywell 635 computer requires 35K words of storage and about 105 s/ω_p^{-1}.

The simulation code has been applied, for the initial study, to both the stable and unstable electrostatically contained plasma column. We have found that the decay rate for stable electrostatic oscillations resulting from the initial distribution

$$f = \frac{e^{-p^2/d^2}}{(2\pi)^{3/4}} e^{-(v_\rho^2+v_\phi^2+v_z^2)/2}(1+\epsilon\cos kz)$$

to be greater than the corresponding homogeneous ($d \to \infty$) result for the axial field. The oscillation frequency of the axial electric field in the inhomogeneous column is also lower than the homogeneous result. The lower frequencies can be attributed to an effective reduction in the axial driving field due to field line loss into the tenuous plasma surrounding the axis. The lower frequencies fit consistently into the usual explanation of Landau damping; and modifications to the axial velocity profile have, at the phase velocity, been recovered at all radial points. These results are also consistent with the usual decay explanation. We believe this is the first time that resonant particle effects have ever been recovered in an inhomogeneous plasma column.

From the frequency versus wavenumber plots for both $E^{\rho}_{N>l}$ and $E^{z}_{N>l}$, we have recovered three oscillation regimes: I) A fast oscillation is predominately axial in character for wavenumbers approaching the inverse Debye length; II) For intermediate wavenumbers, the oscillation becomes dominated by radial motion; and, III) Oscillations for small wavenumbers are slow enough to allow a complete mixing of the oscillations of regimes I) and II).

The axial two stream instability has also been investigated in the inhomogeneous column. Starting with the initial distribution,

$$f = \frac{e^{-p^2/d^2}}{(2\pi)^{3/4}} v_z^2 e^{-(v_\rho^2+v_\phi^2+v_z^2)/2}(1+\epsilon\cos kz)$$

We find the unstable growth rates of the axial electric field are smaller in the inhomogeneous column than the corresponding homogeneous growth rates. We also find that the growth rate decreases as the inhomogeneity increases. It has been shown that if the column is sufficiently inhomogeneous, even the most unstable wavenumber may be completely stabilized in agreement with Harris (1964). In addition, we find the growth rate of the axial field to fall more rapidly for large wavenumbers than the homogeneous results lead one to expect. The source of the discrepancy is the large growth rate of the radial electric field in this region. Energy that produces the growth of the axial field in the homogeneous case is preferentially diverted to unstable radial components in the short wavelength regime.

We have observed the unstable growth of the radial and axial electric fields almost to nonlinear limitation. Due to the small growth rate, we are unable to obtain a clear limiting amplitude. In the long–time regime, some difficulty with accuracy on the radial grid has been experienced. Phenomena near the axis are the last to be affected by this error. A large part of this problem is shown to be due to the minimal number of grid points in addition to the wide variation of the quantities represented on the grid.